ESTIMATING HETEROGENEOUS EFFECTS OF CONTINUOUS EXPOSURES WITH BART

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JSM 2020

August 6, 2020

TEXAS
The University of Texas at Austin

Introduction: Abortion-crime hypothesis

- Donohue and Levitt (2001): legalization of abortion in the US in the 1970s helped lead to a dramatic reduction of crime in the 1980s and 1990s.
- Claim a large negative effect after controlling for socioeconomic variables & state- and year-level fixed effects

THE
QUARTERLY JOURNAL
OF ECONOMICS

Vol. CXVI May 2001 Issue 2

THE IMPACT OF LEGALIZED ABORTION ON CRIME*

JOHN J. DONOHUE III AND STEVEN D. LEVITT

Control variables

Covariate	Description
police	log-police employment per capita
prison	log-prisoner population per capita
gunlaw	indicator variable for presence of concealed weapons law
unemployment	state unemployment rate
income	state log-income per capita
poverty	state poverty rate
afdc15	generosity to Aid to Families with Dependent Children (AFDC), lagged by 15 years
beer	beer consumption per capita

Sensitivity to model specification

- Subsequent studies criticized the functional form of controls
- Belloni et al. (2014) and Hahn et al. (2018) add interactions:
    - state-level controls $\times$ year
    - state-level controls $\times$ year^2
    - state dummies $\times$ year
    - state dummies $\times$ year^2

  After adding these, they claim the causal effect disappears
- Retrospective study by Donohue and Levitt (2019) found that *their predictions from 2001 held up over the next 17 years*
- Woody, Carvalho, and Murray (2020b): adding quadratic trends is the tipping point in negating the causal effect

Our contribution

Present a model which:

(i) Does not require *a priori* parametric specification for controls

(ii) Identifies effect modification by pre-specified moderators

- Detecting unanticipated effect heterogeneity can *generate novel hypotheses* regarding mechanism, e.g. social support

(iii) Gives interpretable summaries of effect modification using method of *posterior summarization*

Methods

Setup

- **Goal:** estimate causal effect of continuous treatment / exposure $Z \in \mathcal{Z} \subseteq \mathbb{R}$ on some outcome Y

- Use potential outcome framework*:

  Compare $Y(Z = z)$ vs. $Y(Z = z')$ for $z, z' \in \mathcal{Z}$

*see, e.g., Imbens and Rubin (2015)

Identifying assumptions

(i) *Consistency*†

$$Z = z \text{ implies } Y = Y(z)$$

(ii) *Weak unconfoundedness*‡

$$Y(z) \perp\!\!\!\perp Z \mid X \text{ for all } z \in \mathcal{Z}$$

(iii) *Positivity*§

$$\pi(z \mid x) > 0 \text{ for all } z \in \mathcal{Z}$$

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†Rubin (1978)
‡Imbens (2000)
§Generalized propensity score, Imbens (2000); Hirano and Imbens (2004)

Causal estimands

- Finite difference average treatment effect (ATE):

$$\text{ATE}_{z',z} = \mathsf{E}[Y(z') - Y(z)]$$

- Dose-response curve:

$$\phi(z) = \mathsf{E}[Y(z)]$$

- Finite difference conditional average treatment effect (CATE):

$$\text{CATE}_{z',z}(x) = \mathsf{E}[Y(z') - Y(z) \mid X = x]$$

Proposed semiparametric model

$$y = \mu(x_{\mathcal{C}}) + \tau(x_{\mathcal{M}}) \cdot z + \varepsilon, \quad \varepsilon \sim \mathrm{N}(0, \sigma^2)$$

- $\mu(\cdot)$ is the *control function*
  $x_{\mathcal{C}}$ is vector of the *control variables*
- $\tau(\cdot)$ is the *exposure moderating function*
  $x_{\mathcal{M}}$ is a vector of *moderators*.
- **Main parametric assumption**:
  y is linear in z with slope determined by $\tau(x_{\mathcal{M}})$

The conditional average treatment effect (CATE) is:

$$\text{CATE}_{z',z}(x) = \tau(x_{\mathcal{M}}) \cdot (z' - z)$$

Proposed semiparametric model

$$y = \mu(x_{\mathcal{C}}) + \tau(x_{\mathcal{M}}) \cdot z + \varepsilon, \quad \varepsilon \sim \mathrm{N}(0, \sigma^2)$$

- $\mu(\cdot)$, $\tau(\cdot)$ modeled using Bayesian additive regression trees[¶]
- Allow for interactions and nonlinearities (no need for *a priori* parametric specification)
- Prior based on Hahn, Murray, and Carvalho (2020), regularize $\tau(\cdot)$ more heavily (shallower trees)

[¶] Chipman, George, and McCulloch (2010); review: Hill, Linero, and Murray (2020)

Application to D&L data

The data

- **Outcome:** y_{st} is the *murder rate* in state s for year t
- **Exposure:** z_{st} is the “effective abortion rate” (D&L, 2001)
  - Lags and weights abortion rates from previous years
- 48 contiguous US states, years 1985–1997 ($N = 624$)
- Denote observations by $i = 1, \ldots, N$

The data

Covariate	Description	Used as control?	Used as moderator?
state	categorical variable for state (contiguous US states; 48 levels)	Yes	Yes
year	numeric value for year (1985–1997, inclusive)	Yes	Yes
police	log-police employment per capita	Yes	No
prison	log-prisoner population per capita	Yes	No
gunlaw	indicator variable for presence of concealed weapons law	Yes	No
unemployment	state unemployment rate	Yes	Yes
income	state log-income per capita	Yes	Yes
poverty	state poverty rate	Yes	Yes
afdc15	generosity to Aid to Families with Dependent Children (AFDC), lagged by 15 years	Yes	Yes
beer	beer consumption per capita	Yes	Yes

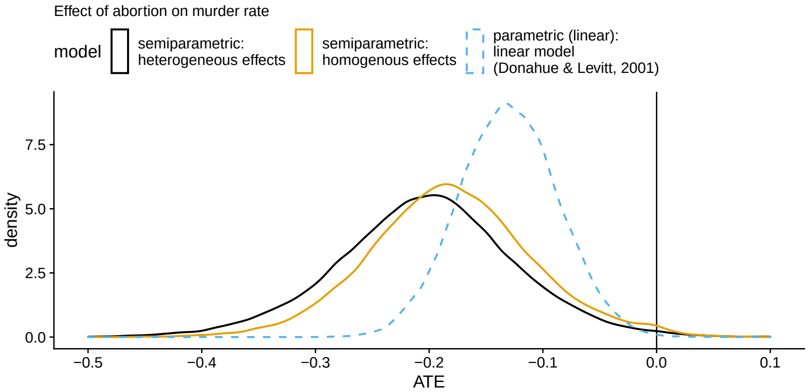
Model definition

$$y = \mu(x_{\mathcal{C}}, s, t) + \tau(x_{\mathcal{M}}, s, t) \cdot z + \varepsilon, \quad \varepsilon \sim \mathrm{N}(0, \sigma^2)$$

Comparison with Donohue and Levitt (2001); Belloni et al. (2014); Hahn et al. (2018); and others:

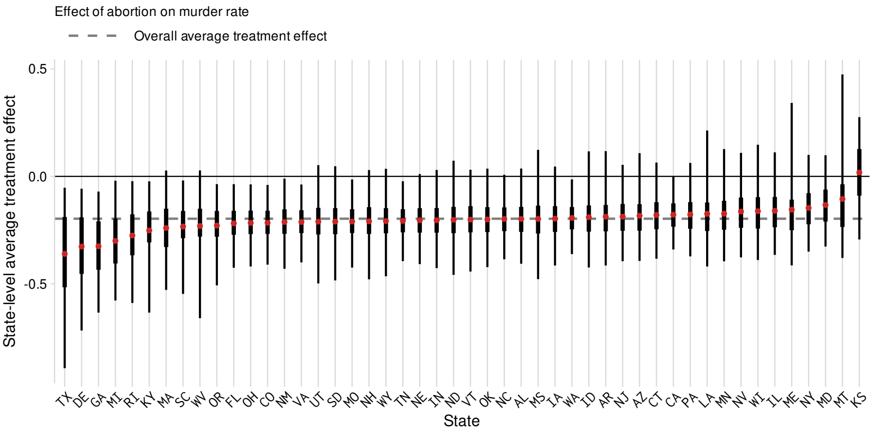
- **Commonality:** Assume linearity of y in z
- **Two departures:**
  - (i) No strict *a priori* parametric specification for controls
  - (ii) Effect heterogeneity through varying slope of treatment effect

ATE estimates



- ATE $= \bar{\tau} = N^{-1} \sum_{i=1}^{N} \tau(x_i)$
- Homogeneous effects model: $\tau(\cdot) \equiv 1$
- Donahue and Levitt (2019), years 1998-2014: ATE = –0.154

State-level ATEs



Characterizing effect heterogeneity

- High degree of heterogeneity between states
- What about heterogeneity driven by moderators?
- Variation in effect across time?

Posterior summary for effect modification

- $\tau(\cdot)$ is nonparametric function, typically difficult to interpret
- We can interpret model through **posterior summarization**‖
- Project $\tau(\cdot)$ down onto a simpler (additive) structure:

$$\tau(\cdot) \approx \gamma(x_i, s_i, t_i) = \bar{\tau} + \sum_{k=1}^{47} b_s \cdot 1(s_i = k) + \sum_{j=1}^{5} h_j(x_{ij}) + h_6(t_i)$$

- Summary communicates treatment effect modification while averaging over possible interactions in $\tau(\cdot)$

‖Woody, Carvalho, and Murray (2020a), in press at *JCGS*

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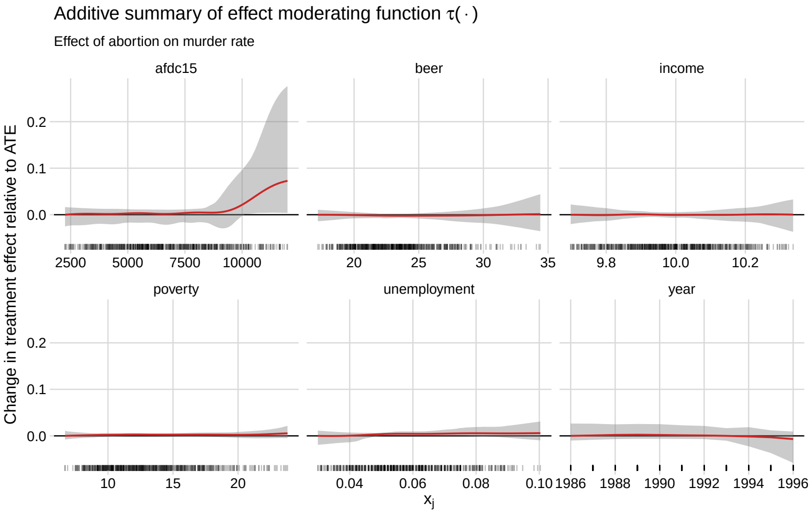
Additive summary



Conclusion

Conclusion

- Strong evidence supporting negative effect of abortion on murder
- Treatment effect heterogeneity
  - Suggestive evidence that `afdc15` mitigates the effect
  - There remains a high degree of unexplained variation in the effect across states
- Reduce replicator degrees of freedom** which can give bias toward false-negatives
- Demonstrate use of modern tools for applied data analyses which are *powerful*, *robust*, and *interpretable*

**Bryan et al., *PNAS* (2019)

More analyses in the paper…

- Posterior summarization for subgroup identification
- Diagnostics of linearity assumption
- Simulation results
- Application to violent crime and property crime
- ArXiv preprint: arxiv.org/abs/2007.09845

Contact

- Session #479 attendee questions:

  Thu Aug 6 at 10:00 AM – 2:00 PM EDT

- Slides: `spencerwoody.github.io/talks`
- ArXiv preprint: `arxiv.org/abs/2007.09845`
- Email: `spencer.woody@utexas.edu`

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Extra slides...

Effective abortion rate

- **Exposure:** z_{st} is the effective abortion rate, e.g.
  - 30% of murders in year t committed by people age 18
  - 70% by age 19, then
  - $\text{EAR}_t = 0.3 \times \text{abortion-rate}_{t-18} + 0.7 \times \text{abortion-rate}_{t-19}$

Diagnostics of linearity assumption

Diagnostics of linearity assumption

Linear effects model:

$$y = \mu(x) + \tau(x) \cdot z + \varepsilon$$

Subtracting out $\mu(x)$ gives:

$$y - \mu(x) = \tau(x) \cdot z + \varepsilon$$

Diagnostics of linearity assumption

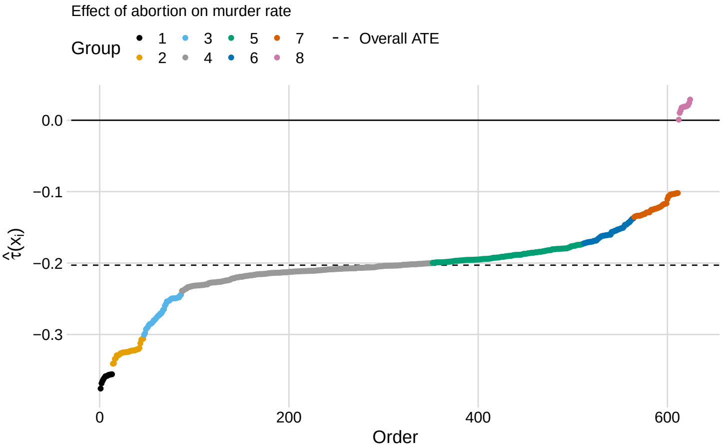
$$y - \mu(x) = \tau(x) \cdot z + \varepsilon$$

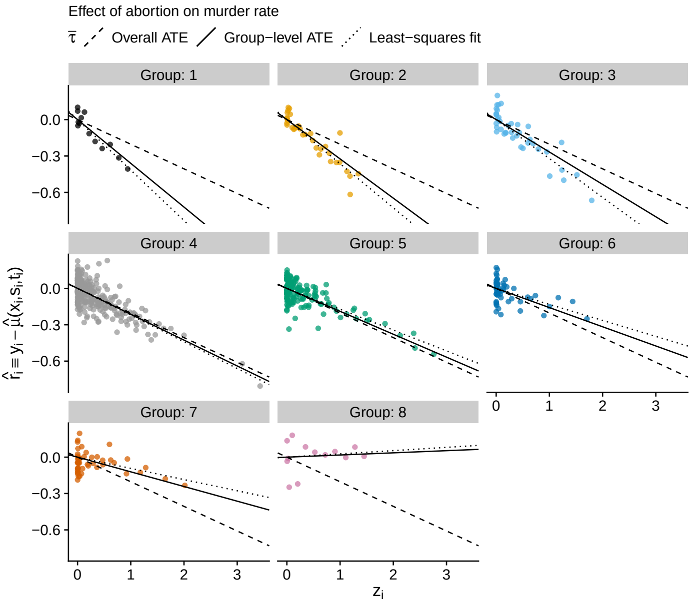
- **Idea:** Combine observations into J disjoint groups g_j such that $\hat{\tau}(x_i) \approx \hat{\tau}(x_{i'})$ for $i, i' \in g_j$, so then

  $$\mathsf{E}[y_i - \hat{\mu}(x_i)] \approx \bar{\tau}_{g_j} \cdot z_i \;\text{ for }\; i \in g_j$$

  where $\bar{\tau}_{g_j} = |g_j|^{-1} \sum_{i \in g_j} \hat{\tau}(x_i)$

- Then plot partial residuals $\hat{r}_i \equiv y_i - \hat{\mu}(x_i)$ against z_i to check for linearity within each group


Effect of abortion on murder rate
Group
1
2
3
4
5
6
7
8
Overall ATE
0.0
−0.1
−0.2
−0.3
$\hat{\tau}(x_i)$
0
200
400
600
Order


Effect of abortion on murder rate
τ̄ Overall ATE Group−level ATE Least−squares fit
Group: 1
Group: 2
Group: 3
Group: 4
Group: 5
Group: 6
Group: 7
Group: 8
0.0
−0.3
−0.6
0 1 2 3
$\hat{r}_i \equiv y_i - \hat{\mu}(x_i, s_i, t_i)$
z_i

Partial dose response curve